UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 2ND FEBRUARY 2012

Organised by the **United Kingdom Mathematics Trust**
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The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

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1. How many of the following four numbers are prime?

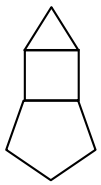
   3 33 333 3333

   A 0 B 1 C 2 D 3 E 4

2. Three positive integers are all different. Their sum is 7. What is their product?

   A 12 B 10 C 9 D 8 E 5

3. An equilateral triangle, a square and a pentagon all have the same side length. The triangle is drawn on and above the top edge of the square and the pentagon is drawn on and below the bottom edge of the square. What is the sum of the interior angles of the resulting polygon?



   A $10 \times 180°$ B $9 \times 180°$ C $8 \times 180°$ D $7 \times 180°$ E $6 \times 180°$

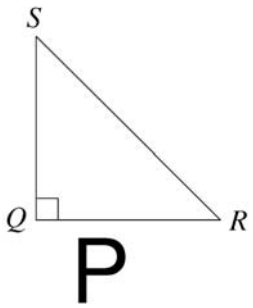
4. All four digits of two 2-digit numbers are different. What is the largest possible sum of two such numbers?

   A 169 B 174 C 183 D 190 E 197

5. How many minutes will elapse between 20:12 today and 21:02 tomorrow?

   A 50 B 770 C 1250 D 1490 E 2450

6. Triangle *QRS* is isosceles and right-angled.
   Beatrix reflects the P-shape in the side *QR* to get an image.
   She reflects the first image in the side *QS* to get a second image.
   Finally, she reflects the second image in the side *RS* to get a third image.
   What does the third image look like?

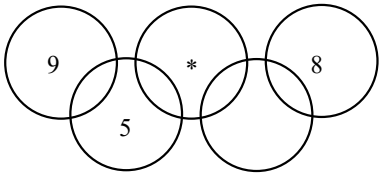


   A B C D E

7. The prime numbers p and q are the smallest primes that differ by 6. What is the sum of p and q?

   A 12 B 14 C 16 D 20 E 28

8. Seb has been challenged to place the numbers 1 to 9 inclusive in the nine regions formed by the Olympic rings so that there is exactly one number in each region and the sum of the numbers in each ring is 11. The diagram shows part of his solution.
   What number goes in the region marked * ?



   A 6 B 4 C 3 D 2 E 1

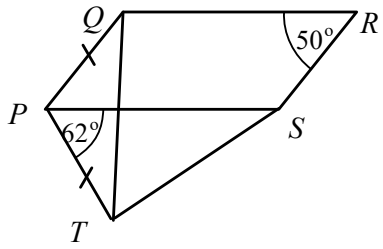
9. Auntie Fi's dog Itchy has a million fleas. His anti-flea shampoo claims to leave no more than 1% of the original number of fleas after use. What is the least number of fleas that will be eradicated by the treatment?

   A 900 000 B 990 000 C 999 000 D 999 990 E 999 999

10. An 'abundant' number is a positive integer N, such that the sum of the factors of N (excluding N itself) is greater than N. What is the smallest abundant number?

A 5 B 6 C 10 D 12 E 15

11. In the diagram, $PQRS$ is a parallelogram; $\angle QRS = 50°$; $\angle SPT = 62°$ and $PQ = PT$.
What is the size of $\angle TQR$?



A 84° B 90° C 96° D 112° E 124°

12. Which one of the following has a different value from the others?

A 18% of £30 B 12% of £50 C 6% of £90 D 4% of £135 E 2% of £270

13. Alex Erlich and Paneth Farcas shared an opening rally of 2 hours and 12 minutes during their table tennis match at the 1936 World Games. Each player hit around 45 shots per minute.
Which of the following is closest to the total number of shots played in the rally?

A 200 B 2000 C 8000 D 12 000 E 20 000

14. What value of x makes the mean of the first three numbers in this list equal to the mean of the last four?

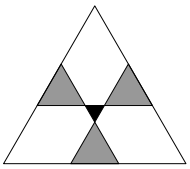
15 5 x 7 9 17

A 19 B 21 C 24 D 25 E 27

15. Which of the following has a value that is closest to 0?

A $\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}$ B $\frac{1}{2}+\frac{1}{3}\div\frac{1}{4}$ C $\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}$ D $\frac{1}{2}-\frac{1}{3}\div\frac{1}{4}$ E $\frac{1}{2}-\frac{1}{3}\times\frac{1}{4}$

16. The diagram shows a large equilateral triangle divided by three straight lines into seven regions. The three grey regions are equilateral triangles with sides of length 5 cm and the central black region is an equilateral triangle with sides of length 2 cm.
What is the side length of the original large triangle?



A 18 cm B 19 cm C 20 cm D 21 cm E 22 cm

17. The first term of a sequence of positive integers is 6. The other terms in the sequence follow these rules:

if a term is even then divide it by 2 to obtain the next term;

if a term is odd then multiply it by 5 and subtract 1 to obtain the next term.

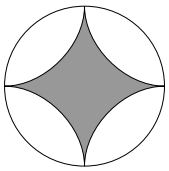
For which values of n is the n th term equal to n?

A 10 only B 13 only C 16 only D 10 and 13 only E 13 and 16 only

18. Peri the winkle starts at the origin and slithers anticlockwise around a semicircle with centre (4, 0). Peri then slides anticlockwise around a second semicircle with centre (6, 0), and finally clockwise around a third semicircle with centre (3, 0).
Where does Peri end this expedition?

A (0, 0) B (1, 0) C (2, 0) D (4, 0) E (6, 0)

19. The shaded region shown in the diagram is bounded by four arcs, each of the same radius as that of the surrounding circle. What fraction of the surrounding circle is shaded?



A $\frac{4}{\pi} - 1$ B $1 - \frac{\pi}{4}$ C $\frac{1}{2}$ D $\frac{1}{3}$ E it depends on the radius of the circle

20. A rectangle with area 125 cm^2 has sides in the ratio 4:5. What is the perimeter of the rectangle?

A 18 cm B 22.5 cm C 36 cm D 45 cm E 54 cm

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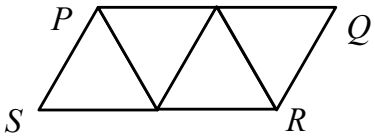
21. The parallelogram $PQRS$ is formed by joining together four equilateral triangles of side 1 unit, as shown.
What is the length of the diagonal SQ?

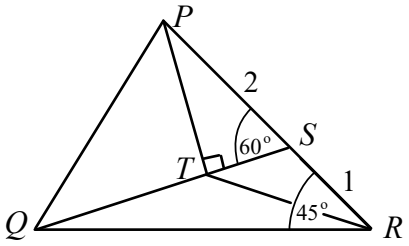


A $\sqrt{7}$ B $\sqrt{8}$ C 3 D $\sqrt{6}$ E $\sqrt{5}$

22. What is the maximum possible value of the median number of cups of coffee bought per customer on a day when Sundollars Coffee Shop sells 477 cups of coffee to 190 customers, and every customer buys at least one cup of coffee?

A 1.5 B 2 C 2.5 D 3 E 3.5

23. In triangle PQR, $PS = 2$; $SR = 1$; $\angle PRQ = 45°$; T is the foot of the perpendicular from P to QS and $\angle PST = 60°$.
What is the size of $\angle QPR$?



A 45° B 60° C 75° D 90° E 105°

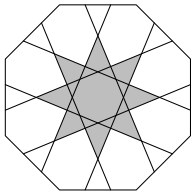
24. All the positive integers are written in the cells of a square grid. Starting from 1, the numbers spiral anticlockwise. The first part of the spiral is shown in the diagram.

					…	32	31	
		17	16	15	14	13	30	
		18	5	4	3	12	29	
		19	6	1	2	11	28	
		20	7	8	9	10	27	
		21	22	23	24	25	26	

What number will be immediately below 2012?

A 1837 B 2011 C 2013 D 2195 E 2210

25. The diagram shows a ceramic design by the Catalan architect Antoni Gaudi. It is formed by drawing eight lines connecting points which divide the edges of the outer regular octagon into three equal parts, as shown.
What fraction of the octagon is shaded?



A $\frac{1}{5}$ B $\frac{2}{9}$ C $\frac{1}{4}$ D $\frac{3}{10}$ E $\frac{5}{16}$